upstream with each sweep. These results imply that the real flow is turbulent and has a shorter recirculation region, which is consistent with experiments. It also suggests that there is little merit in expending effort to calculate the large laminar separation bubbles that would be obtained with larger reduced angles. This observation is likely to be independent of the use of interactive boundary layer or Navier-Stokes procedures.

Acknowledgment

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Analysis of Multi-Element Airfoils by a Vortex Panel Method

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Nomenclature

$A_{i,j}$ = influence coefficient matrix
b_i = right-hand side of Eq. (2) (function of σ)
C_p = pressure coefficients
C = multi-element airfoil contour
F = function entering the Cauchy-type integral, Eq. (1)
f = jump of F across airfoil contour
$m(l)$ = number of points on lth component
N = number of points defining multi-element system
N_c = number of components
u = tangential component of velocity
v = normal component of velocity
(x,y) = coordinate of airfoil
Z = complex position vector
α = angle of attack
γ = vortex strength
σ = source strength
ϕ = angle between exterior normal to C and real axis
θ = trailing-edge angle

Subscripts

i,j = panel indices
L = lower surface
U = upper surface

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Introduction

SEVERAL methods exist for the analysis of single or multi-element airfoils in potential flows.[1-7] Most of these methods have one or more of the following drawbacks: use of multiple singularities either to alleviate the problem of pressure oscillations near trailing edges of thin airfoils, or to represent the thickness and lifting effects for finite-thickness lifting airfoils;[1-4] use of panel method in a mapped plane when trailing edge is cusped;[5] need to represent airfoil by large number of points to get acceptable results.[6] Also, most of the methods output the pressure coefficients at panel midpoints, which in fact are not the points on the given contour.

In the present Note, we describe a "vortex alone" formulation for the analysis of multi-element airfoils, which does not suffer from any of the problems listed above. Versatility of this formulation is demonstrated by analyzing thin or cusped airfoils and airfoils with two, three, or four elements for which exact results are available.

Method of Analysis

Let $Z = x + iy$ be the complex position vector in the airfoil plane (Fig. 1). Identifying $F(Z)$ with the complex disturbance velocity in the Cauchy-type of integral

$$F(Z) = \frac{1}{2\pi i}\int \frac{f(Z')}{Z' - Z}\,\mathrm{d}Z' \tag{1}$$

where $f(Z')$ is jump of function F across the contour C, by writing

$$f(Z') = -[\sigma(Z') + i\gamma(Z')]e^{-i\phi(Z')}$$

it can be shown that the normal and tangential components are given by

$$v(Z) = \mathrm{Re}\,\{[e^{-i\alpha} + F^{-}(Z)]e^{i\phi(z)}\} = \sigma(Z)$$

$$u(Z) = -\mathrm{Im}\,\{[e^{-i\alpha} + F^{-}(Z)]e^{i\phi(z)}\} = -\gamma(Z)$$

respectively, where α is freestream incidence. Here, nonzero values of σ can be used to simulate the displacement effects of the boundary layer. Zero value of σ implies tangential inviscid flow and leads to a vortex alone formulation for the potential flow over multi-element airfoils. The trailing-edge Kutta con-

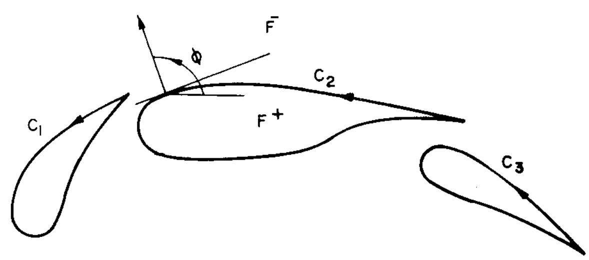

Fig. 1 Complex Z-plane.

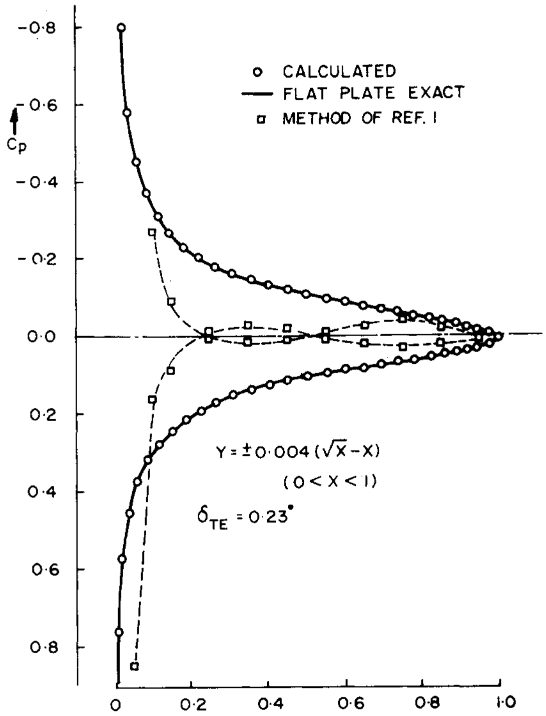

Fig. 2 ***C_p* distribution on an extremely thin airfoil.**

dition of finite trailing-edge velocities becomes

$$\sigma_L + i\gamma_L = -(\sigma_u + i\gamma_u)\, e^{i\theta}$$

where θ is the trailing-edge angle. Now, using Eq. (1), the complex disturbance velocity at any field point Z can be written as

$$F^-(Z) = \sum_{l=1}^{N_c} \sum_{j=m(l-1)+1}^{m(l)} \Delta_j F^-(Z) \tag{2}$$

where

$$\Delta_j F^-(Z) = \frac{1}{2\pi i} \int_{Z_j}^{Z_{j+1}} \frac{F(Z')}{Z' - Z}\, \mathrm{d}Z'$$

is the contribution of the jth panel. If we assume a linear variation of f in Z, Eq. (2) along with Kutta condition can be written as

$$\sum_{l=1}^{N_c} \sum_{j=m(l-1)+1}^{m(l)} A_{ij}\gamma_j = b_i, \qquad i = 1,2,\ldots,N + N_c$$

which is an overdetermined set of equations. If values of σ are zero, the system of equations, Eq. (2), is closed and leads to an inviscid tangential flow analysis. Also, it may be noted that the γ are the value of vortex strengths γ_j at the panel corner points. Once the strengths of γ are determined, since σ are known, by Bernoulli's equation the pressure coefficient is given by

$$C_p(Z_j) = 1 - (\sigma_j^2 + \gamma_j^2)$$

For a cusped trailing edge, the slopes of the last top and bottom trailing-edge panels become very nearly equal, and the matrix coefficient for the effect of the top panel at the bottom control point or vice versa at this location should approach $\frac{1}{4}\pi$ in the limit. Once this limiting behavior is taken into account properly, the surface vortex method poses no problem for cusped or thin trailing edges.

Examples, Results, and Discussion

Plotted in Fig. 2 are the computational results for a 0.2%-thick airfoil at an $\alpha = 3$, using 64 surface panels and the corresponding exact solutions for a flat plate at same α. The two results are indiscernible. Plotted in the same figure are the results of Ref. 1, which uses an internal doublet singularity method. Oscam[2] overcomes the problem of trailing-edge pressure oscillations of Ref. 1 by distributing linearly varying vorticity distribution on the mean line with constant source flat panels on the actual surface. The results of current method clearly indicate that the distribution of vorticity alone on the actual contour can correctly predict the pressures, even on extremely thin profiles. There is no need to resort to different type of singularities to handle these thin profiles.

The second example involves the application of the present method to Williams two-element configuration A. From the pressure curve (Fig. 3), it may be seen that the vortex panel method correctly predicts suction peaks on both the airfoil components. From Table 1, it can be observed that the present method predicts the lift forces with better accuracy than the source panel method.[6] But a more important point to be noted is that the resultant drag component of a two-element system is closer to zero in the current method.

Similarly, the final two examples chosen for evaluating the method described are the three- and four-component airfoils of Ref. 8 for which exact solutions are known. The computed and exact pressure coefficients are plotted in Fig. 4. The results of current program are in good agreement with the exact results.

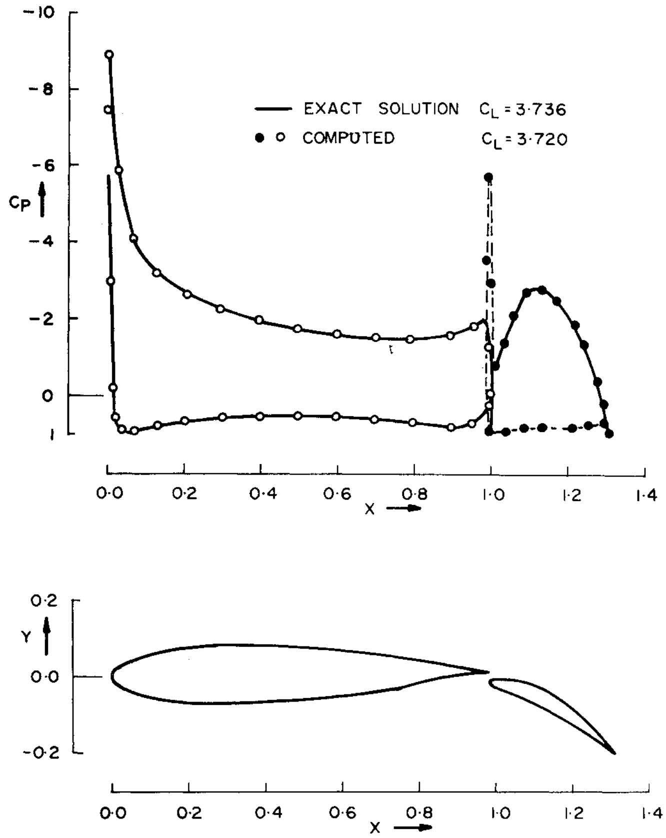

Fig. 3 Pressure distributions on the Williams two-element airfoil: *A* (δ_f = 30 deg), α = 0.0.

Table 1

	Main airfoil		Flap		Total	
Williams—*A*	cl	ca	cl	ca	cl	ca
Exact	2.9065	−0.3839	0.8302	0.3838	3.7367	−0.0001
Amos	2.8700	−0.3752	0.7984	0.3559	3.6684	−0.0193
Current	2.9000	−0.3870	0.8203	0.3830	3.7203	−0.004

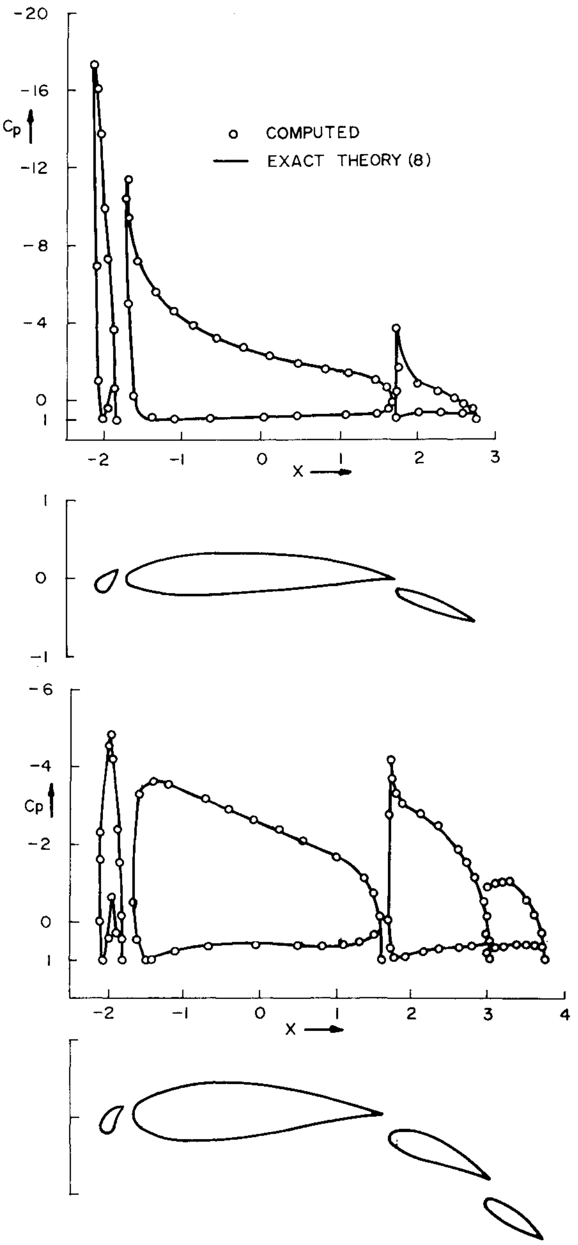

Fig. 4 Comparison of C_p distributions for three- and four-element airfoils.

Conclusions

The distributed vortex singularity method using flat panels with linearly varying vortex strength for obtaining inviscid solution on single or multi-element airfoils is an accurate and stable method. Once the limiting behavior of trailing-edge panels is correctly accounted for, the method can analyze airfoils of arbitrary thickness and camber, airfoils with finite trailing-edge angle, and airfoils with cusped trailing edges. Also, there are no problems faced in analyzing extremely thin airfoils. Another important advantage of the current formulation is that the pressure coefficients are computed at the contour input points, whereas most methods output the C_P at panel midpoints, which in fact are not points on the contour. The method has been checked for a very large number of cases, and the results show that as few as 20 panels are enough to compute pressure coefficients over a single-component airfoil. A viscous version of the method is available in Ref. 9.

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Unsteady Pressure Distribution Over a Pitching Airfoil

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Introduction

THE problem of unsteady aerodynamic loads over airfoils has been intensively studied, but some published contradictory results still raise the question of whether a potential model is valid or not. The general frame of the present investigation is the unsteady loads over airplane wings during flight in turbulence. The main unsteady parameter is the reduced frequency, $k = 2\pi fc/U_\infty$, and in the present case the range of interest is $0<k<1.2$. The amplitude of the angle-of-attack fluctuations is small enough to ensure that the flow remains attached, at least over the main part of the airfoil. However, the unsteady mechanism of the separation of the boundary layers at the trailing edge, and consequently the modeling of the Kutta condition in inviscid numerical codes, remains an open subject.

A lot of experimental work has been done concerning a possible violation of the Kutta condition.[1-4] As pointed out by Telionis and Poling,[4] most of these investigations examine pressure distributions, especially the differential pressure loading at the trailing edge. Satyanarayana and Davis[1] observe a nonzero loading (first harmonic of the unsteady pressure) at $k = 1.23$ on a pitching airfoil. They conclude that the failure of the Kutta condition is obvious.

This result seems questionable. First, the differential pressure loading must vanish physically at the trailing edge, at least in a real viscous fluid. The question is rather the rate at

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